Lesson 4: The Unit Circle

Now we are ready to explore trigonometric functions. We will use the unit circle approach. The **unit circle** that we will develop is the most useful tool in trigonometry. It provides an easy way to know and recall trigonometric values of the most popular angles. To be successful in this class, and then later in calculus and beyond, you must understand it and **memorize it**!

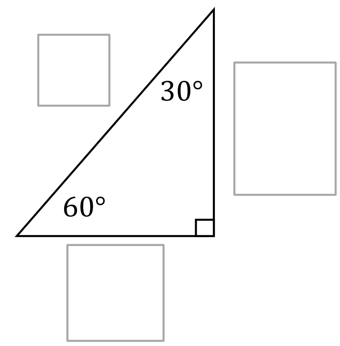
A Geometry Connection

First, let's remember the important ratio relationships for two famous triangles.

30°-60°-90° Triangle

The lengths of the legs of the triangle correspond to the angles 30°: 60°: 90° as, $x : x\sqrt{3} : 2x$ respectively. In the figure at right,

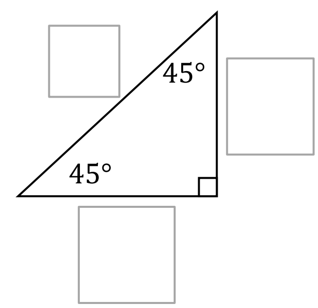
Name the lengths of the sides if the hypotenuse is 1.



45°-45°-90° Triangle

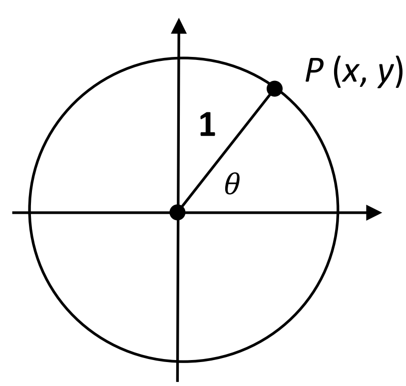
The length of the legs of the triangle correspond to the angles 45°: 45°: 90° as $x : x : x\sqrt{2}$, respectively. In the figure at right,

Name the lengths of the sides if the hypotenuse is 1.



The Unit Circle

The unit circle is a circle of radius 1 centered at the origin.



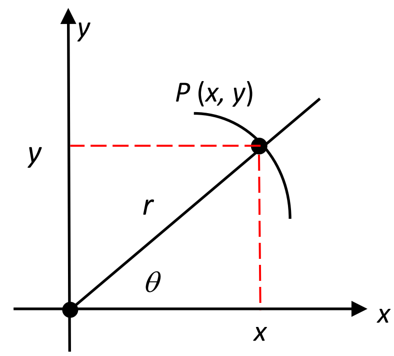
Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and

$$r = \sqrt{x^2 + y^2} \neq 0$$

EX #1: Use the figure at right, to name the six basic trigonometric functions in terms of *x, y*, and *r*, where *r* is the radius of a circle.

$\sin\theta =$	$\csc\theta =$
$\cos\theta =$	$\sec\theta =$
$\tan\theta =$	$\tan\theta =$

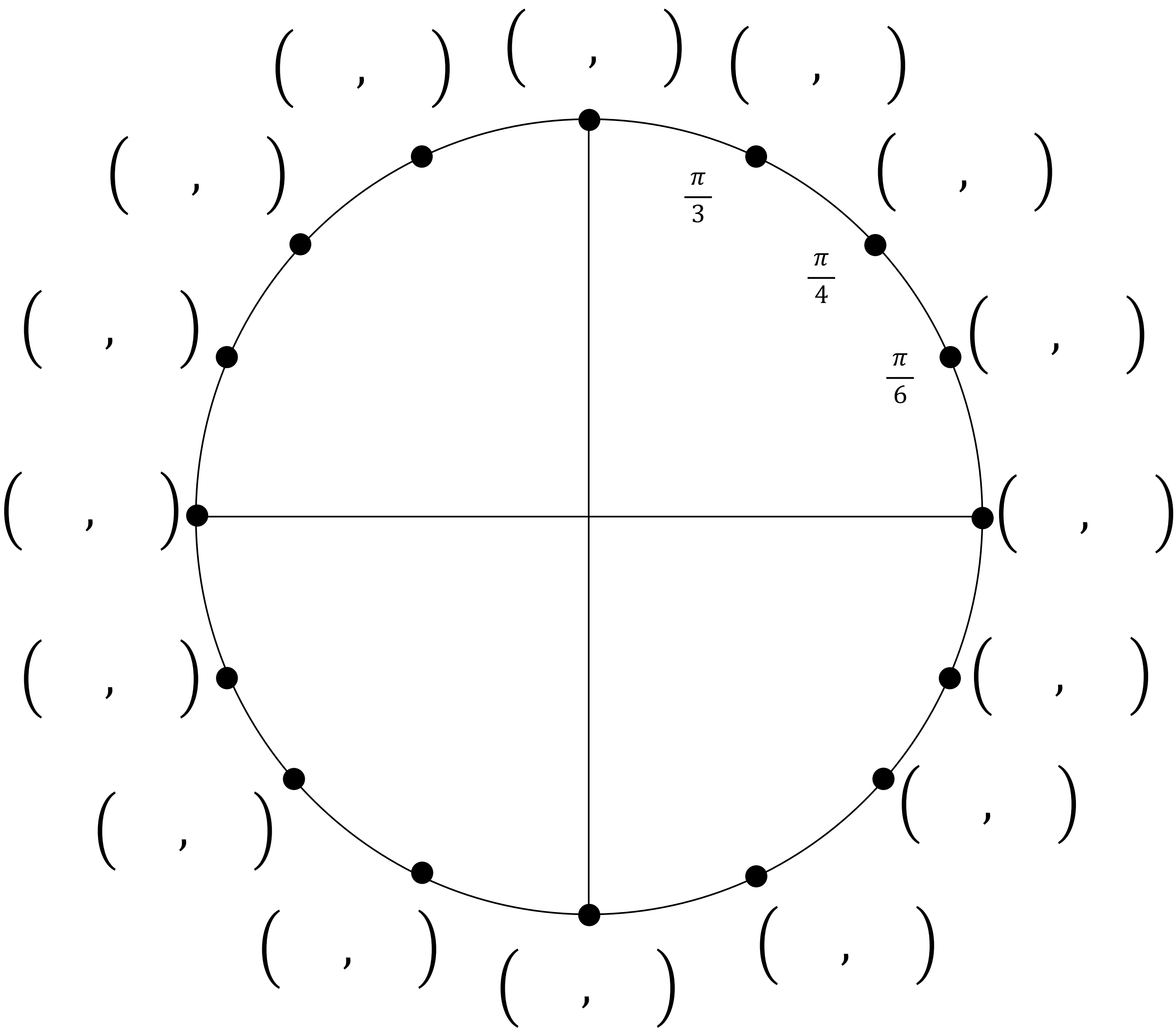


$$r = \sqrt{x^2 + y^2}$$

. . . Where Trigonometric Functions are Positive . . .

Function	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
Sine/Cosecant				
Cosine/Secant				
Tangent/ Cotangent				

The 16-Point Unit Circle



Finding Exact Values of the Trigonometric Functions

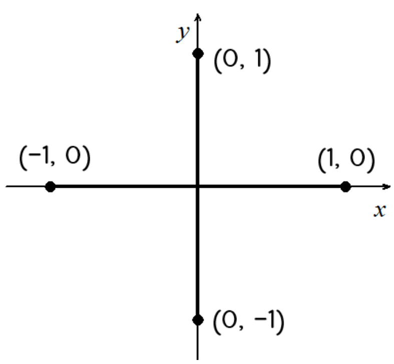
EX #2: Summarize the exact values for 30°, 60°, and 90° angles (rationalize denominators).

θ Degrees	θ Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
30°	$\frac{\pi}{6}$						
45°	$\frac{\pi}{4}$						
60°	$\frac{\pi}{3}$						

Quadrantal Angles

A quadrantal angle is an angle whose terminal side lies along one of the coordinate axes.

EX #3: Evaluate the six trig functions at the four quadrant angles.



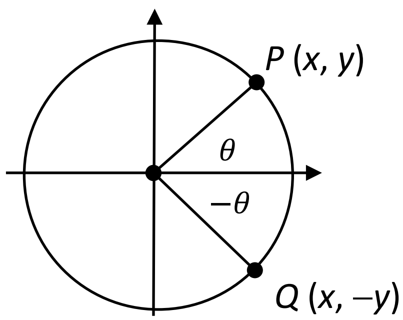
Note:

There is no need to memorize the table, simply draw the angle and apply the definition for any given function.

θ degrees	0°	90°	180°	270°
radians	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$				
$\cos\theta$				
$\tan\theta$				
$\csc\theta$				
$\sec\theta$				
$\cot\theta$				

Odd vs. Even Trigonometric Functions

Remember that a function f is even if $f(-x) =$ _______________ for all x in the domain of f and a function f is odd if $f(-x) =$ ________________ for all x in the domain of f.

	Even Trig Functions	Odd Trig Functions
	$\cos(-\theta) = \cos(\theta)$ $\sec(-\theta) = \sec(\theta)$	$\sin(-\theta) = -\sin(\theta)$ $\csc(-\theta) = -\csc(\theta)$ $\tan(-\theta) = -\tan(\theta)$ $\cot(-\theta) = -\cot(\theta)$

EX #4: Find the exact value of each of the following.

A. $\cos(-60°)$

B. $sin\left(-\frac{\pi}{2}\right)$

C. $tan\ (-\pi)$

D. $\cot\left(-\frac{11\pi}{2}\right)$