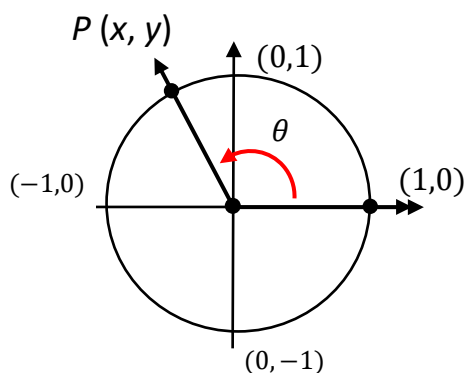


Lesson 5: Properties of the Trigonometric Functions

To be successful in the study of calculus it is important to know some of the basic properties that occur with the six trigonometric functions

Ratios on the Unit Circle



EX #1: In the sketch, at left, θ is in standard position and $P(x, y)$ is a point on the unit circle that corresponds to θ . Complete the following in terms of (x, y) :

$$\sin \theta =$$

$$\cos \theta =$$

$$\tan \theta =$$

$$\csc \theta =$$

$$\sec \theta =$$

$$\cot \theta =$$

Domain and Range of the Trigonometric Functions

Function	Undefined on $[0, 2\pi)$	Domain	Range
sine			
cosine			
tangent			
cosecant			
secant			
cotangent			

Periodic Functions and Finding Exact Values

The behavior of sine and cosine repeats after one revolution. This repetitive (or cyclic) pattern is called **periodic**.

Therefore, one cycle for sine and cosine has a period _____, while the tangent function has a period _____.

EX #2: Evaluate without a calculator by using the period of the function and values on the unit circle.

A. $\sin\left(\frac{23\pi}{3}\right)$

B. $\cos\left(\frac{17\pi}{4}\right)$

C. $\tan\left(-\frac{9\pi}{2}\right)$

D. $\sin\left(-\frac{29\pi}{6}\right)$

Trigonometric Identities

Look back at the ratios from Example 1, can you name the reciprocal identities and quotient identities in terms of a trigonometric function?

Reciprocal Identities

$\csc \theta =$

$\sec \theta =$

$\cot \theta =$

Quotient Identities

$\tan \theta =$

$\cot \theta =$

Finding Exact Values Given a Terminal Point

When given the terminal point of an angle rotated on a coordinate plane, you should create a right triangle by dropping a perpendicular line back to the x-axis and use the Pythagorean Theorem to find the hypotenuse.

EX #3: Find the exact values of each of the six trigonometric functions of an angle θ if $(-2, 5)$ is a point on its terminal side.

Using Fundamental Identities

EX #4: Given $\sin \theta = \frac{2\sqrt{13}}{13}$ and $\cos \theta = \frac{3\sqrt{13}}{13}$, find the exact values of the four remaining trigonometric functions of θ using identities.

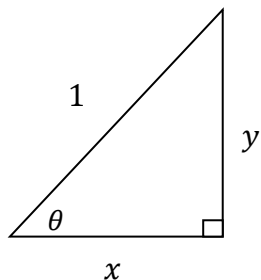
EX #5: Use properties of the trigonometric functions to find the exact value of each expression.

A: $\tan 50^\circ - \frac{\sin 50^\circ}{\cos 50^\circ}$

B: $(\cos 400^\circ)(\sec 40^\circ)$

C: $\left(\sin \frac{\pi}{6}\right)\left(\csc \frac{13\pi}{6}\right)$

Pythagorean Identities



Pythagorean Identities

If $P = (x, y)$ is the point on the unit circle that corresponds to the angle θ , then the equation of the unit circle is _____.

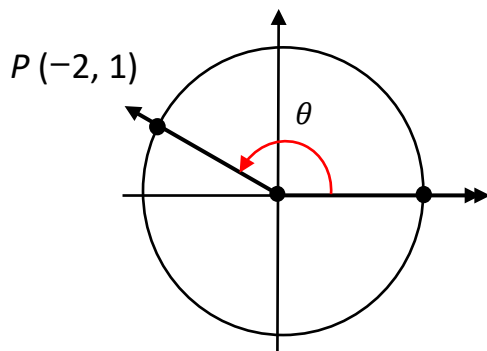
We know that $y = \sin \theta$ and $x = \cos \theta$, so we can substitute these into our equation to give us

There are two more Pythagorean Identities we can derive ...

Divide each side of the equation by $\cos^2 \theta$:

Divide each side of the equation by $\sin^2 \theta$:

Using Identities



EX #6: Given that $\tan \theta = -\frac{1}{2}$ and $\sin \theta > 0$, find the exact value of each of the remaining five trigonometric functions.