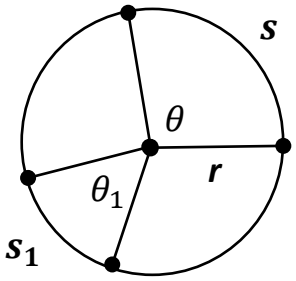
Lesson 3: Arc Length and Circular Motion

In geometry, we found if given a central angle and the length of the radius, we could set up a proportion to find the length of an arc in a circle. ***The arc length is a fraction of the total circumference, and the central angle is a fraction of a full revolution.***

Arc Length Formula

Likewise, from geometry we know that the ratio of the measures of the angles equals the ratio of the corresponding lengths of the arc subtended by these angles.



That is, $\frac{\theta}{\theta_1} = \frac{s}{s_1}$

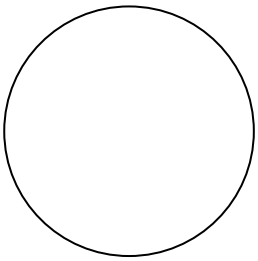
If we let $\theta_1 = 1$ radian, then $s_1 = r$ from our previous definition of a radian. From here we can find a formula for arc length.

Arc Length Formula

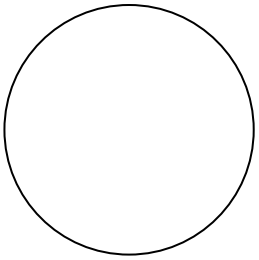
$$s = r\theta$$

(where θ is in radians, never degrees!)

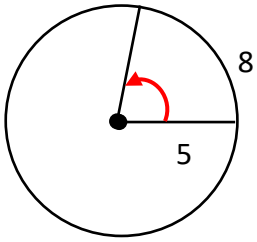
EX #1: Given a central angle of 60° and a radius of 8 cm, find the length of the intercepted arc.



EX #2: If a 100° arc of a circle has a length of 9 inches, to the nearest inch, what is the radius of the circle?



EX #3: Find the angle in radians, then convert to the nearest degree.



Circular Motion

Linear Velocity (speed)

Have you ever thought about math while out for a bicycle ride? Probably not! You know how to find your average speed by calculating distance traveled divided by the time from long ago (rate = distance/time or $R = D/T$). In trigonometry, we will call this **linear (speed) velocity** and rename our variables using v for velocity (rate) and s (arc length) for distance.

Linear Velocity Formula

$$v = \frac{\Delta s}{\Delta t} \Rightarrow \Delta s = v(\Delta t)$$

EX #4: Dimensional Analysis

If you travel in a car at 65mph, how far did you go in 10 seconds?

Angular Velocity (speed)

$$\omega = \frac{\Delta \theta}{\Delta t} \Rightarrow \Delta \theta = \omega(\Delta t)$$

Now let's look at another type of velocity while we ride our bicycle. Think about how fast the wheels are rotating. This is called **angular (speed) velocity**. The Greek letter omega "ω" is the accepted variable used for angular speed. This is the rate that measures changes in the wheel's central angle, θ, over time.

Angular Velocity Formula

EX #5: If a central angle spins 15π radians in 8 seconds, how fast is the angle spinning per second? (Give your answer to nearest thousandths.)

Units and Converting Between Linear and Angular

Velocity Summary

Linear velocity has dimensions of length per unit of time. Examples, such as, **feet per second, miles per hour, meters per minute**. (*The numerator units are measures of length.*)	
Angular velocity has dimensions of angle measure (radians) per unit of time. Examples, such as **revolutions per minute, radians per second, degrees per hour.** (*The numerator units are angle measurements.*)	

Converting Units of Angular Velocity

EX #6: Convert 60 rpm (revolutions per minute)

A. to radians per minute	**B.** to degrees per second

Important Relationships between Linear and Angular Speed

We can use the previous formulas to find a relationship between linear velocity (v) and angular velocity (ω).

Linear Velocity:

$v = \frac{\Delta s}{\Delta t}$ Linear velocity formula

$v = \frac{r\Delta\theta}{\Delta t}$ Arc formula

$v = r\left(\frac{\Delta\theta}{\Delta t}\right)$ Factoring

$v = r\omega$ Angular velocity formula

Linear Velocity = r • Angular Velocity

$$v = r \cdot \omega$$

Area and Distance

Area of a Sector

$$A = \frac{1}{2}r^2\theta$$

(where θ is in radians, never degrees)

EX #7: Find the area of the sector of a circle of radius 2 feet formed by an angle of 50°. Round the answer to two decimal places.

Finding Distance Between Two Cities

EX #8: Memphis, Tennessee is located at 35°9′ north latitude and New Orleans, Louisiana is at 29°57′north latitude. Assuming that the radius of the Earth is 3960 miles, and the cities lie along the same longitude, find the distance between the two cities.

Free-Response Question

EX #9: Some Earthly Discoveries

A: Compute the length in feet of an angle measure of 1 minute on the Earth, given that the radius of the Earth is 3960 miles.

B: What is the length of an angle of one second on the Earth?