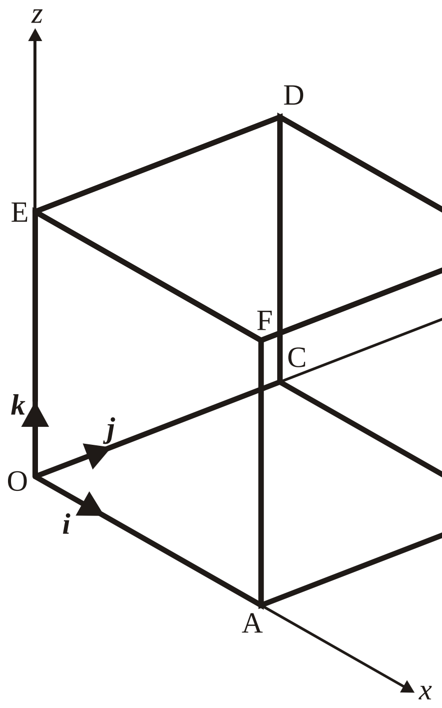
VECTORS - IB SAMPLE PROBLEMS

1. The diagram shows a cube, OABCDEFG where the length of each edge is 5cm. Express the following vectors in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.



(a) $\overrightarrow{\mathrm{OG}}$;

(b) $\overrightarrow{\mathrm{BD}}$;

(c) $\overrightarrow{\mathrm{EB}}$.

(Total 6 marks)

2. Consider the vectors $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = 4\boldsymbol{i} + \boldsymbol{j} - p\boldsymbol{k}$.

(a) Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ find the value of p.

(b) Given that $q|\mathbf{u}| = 14$, find the value of q.

(Total 6 marks)

3. The line L passes through the points A (3, 2, 1) and B (1, 5, 3).

(a) Find the vector $\overrightarrow{\mathrm{AB}}$.

(b) Write down a vector equation of the line L in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(Total 6 marks)

4. The line L_1 is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

(Total 6 marks)

5. Two lines L_1 and L_2 have these vector equations.

$L_1 : \boldsymbol{r} = 2\boldsymbol{i} + 3\boldsymbol{j} + t(\boldsymbol{i} - 3\boldsymbol{j})$
$L_2 : \boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} + s(\boldsymbol{i} - \boldsymbol{j})$

The angle between L_1 and L_2 is θ. Find the cosine of the angle θ.

(Total 6 marks)

6. Consider the points A (1, 5, 4), B (3, 1, 2) and D (3, k, 2), with (AD) perpendicular to (AB).

(a) Find

(i) $\overrightarrow{\text{AB}}$;

(ii) $\overrightarrow{\text{AD}}$, giving your answer in terms of k. **(3)**

(b) Show that $k = 7$. **(3)**

The point C is such that $\overrightarrow{\text{BC}} = \frac{1}{2}\overrightarrow{\text{AD}}$.

(c) Find the position vector of C.
(4)

(d) Find cos $\text{A}\hat{\text{B}}\text{C}$. **(3)**
(Total 13 marks)

7. In this question, distance is in metres, time is in minutes.

Two model airplanes are each flying in a straight line.

At 13:00 the first model airplane is at the point (3, 2, 7). Its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$.

(a) Find the speed of the model airplane.
(2)

At 13:00 the second model airplane is at the point (– 5, 10, 23). After two minutes, it is at the point (3, 16, 39).

(b) Show that its position vector after t minutes is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 23 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$.
(3)

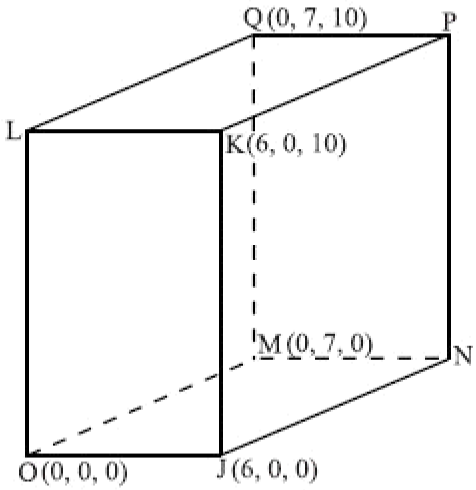
(c) The airplanes meet at point Q.

(i) At what time do the airplanes meet?

(ii) Find the position of Q.
(6)

(d) Find the angle θ between the paths of the two airplanes.
(6)
(Total 17 marks)

8. The diagram below shows a cuboid (rectangular solid) OJKLMNPQ. The vertex O is (0, 0, 0), J is (6, 0, 0), K is (6, 0, 10), M is (0, 7, 0) and Q is (0, 7, 10).



(a) (i) Show that $\overrightarrow{JQ} = \begin{pmatrix} -6 \\ 7 \\ 10 \end{pmatrix}$.

(ii) Find $\overrightarrow{MK}$.

(2)

(b) An equation for the line (MK) is $\boldsymbol{r} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + s\begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix}$.

(i) Write down an equation for the line (JQ) in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Find the acute angle between (JQ) and (MK).

(9)

(c) The lines (JQ) and (MK) intersect at D. Find the position vector of D.

(5)

(Total 16 marks)

9. Points P and Q have position vectors $-5\boldsymbol{i} + 11\boldsymbol{j} - 8\boldsymbol{k}$ and $-4\boldsymbol{i} + 9\boldsymbol{j} - 5\boldsymbol{k}$ respectively, and both lie on a line L_1.

(a) (i) Find $\overrightarrow{PQ}$.

(ii) Hence show that the equation of L_1 can be written as

$$\boldsymbol{r} = (-5 + s)\,\boldsymbol{i} + (11 - 2s)\,\boldsymbol{j} + (-8 + 3s)\,\boldsymbol{k}.$$

(4)

The point R $(2, y_1, z_1)$ also lies on L_1.

(b) Find the value of y_1 and of z_1.

(4)

The line L_2 has equation $\boldsymbol{r} = 2\boldsymbol{i} + 9\boldsymbol{j} + 13\boldsymbol{k} + t\,(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$.

(c) The lines L_1 and L_2 intersect at a point T. Find the position vector of T.

(7)

(d) Calculate the angle between the lines L_1 and L_2.

(7)
(Total 22 marks)