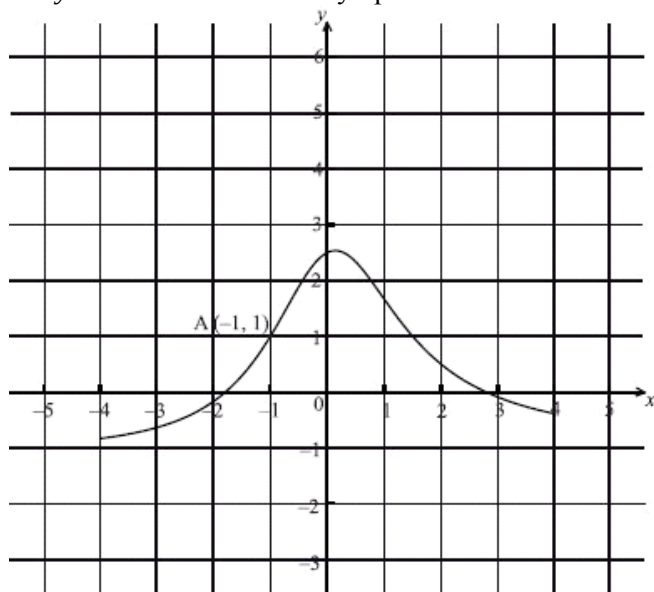


Functions – IB Sample Problems

1. Two weeks after its birth, an animal weighed 13 kg. At 10 weeks this animal weighed 53 kg. The increase in weight each week is constant.
- (a) Show that the relation between y , the weight in kg, and x , the time in weeks, can be written as $y = 5x + 3$ (2)
 - (b) Write down the weight of the animal at birth. (1)
 - (c) Write down the weekly increase in weight of the animal. (1)
 - (d) Calculate how many weeks it will take for the animal to reach 98 kg. (2)
- (Total 6 marks)
2. The functions $f(x)$ and $g(x)$ are defined by $f(x) = e^x$ and $g(x) = \ln(1 + 2x)$.
- (a) Write down $f^{-1}(x)$.
 - (b) (i) Find $(f \circ g)(x)$.
 - (ii) Find $(f \circ g)^{-1}(x)$. (Total 6 marks)
3. Let $f(x) = \sqrt{x+4}$, $x \geq -4$ and $g(x) = x^2$, $x \in \mathbb{R}$.
- (a) Find $(g \circ f)(3)$.
 - (b) Find $f^{-1}(x)$.
 - (c) Write down the domain of f^{-1} . (Total 6 marks)
4. (a) Given that $(2^x)^2 + (2^x) - 12$ can be written as $(2^x + a)(2^x + b)$, where $a, b \in \mathbb{Z}$, find the value of a and of b .
- (b) Hence find the **exact** solution of the equation $(2^x)^2 + (2^x) - 12 = 0$, and explain why there is only one solution. (Total 6 marks)
5. The population of a city at the end of 1972 was 250 000. The population increases by 1.3% per year.
- (a) Write down the population at the end of 1973.
 - (b) Find the population at the end of 2002. (Total 6 marks)
6. The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.
- (a) Write f in the form $f(x) = 3(x - h)^2 - k$.
 - (b) The graph of f is translated 3 units in the positive x -direction and 5 units in the positive y -direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x - p)^2 + q$. (Total 6 marks)
7. Find the **exact** value of x in each of the following equations.
- (a) $5^{x+1} = 625$
 - (b) $\log_a(3x + 5) = 2$ (Total 6 marks)

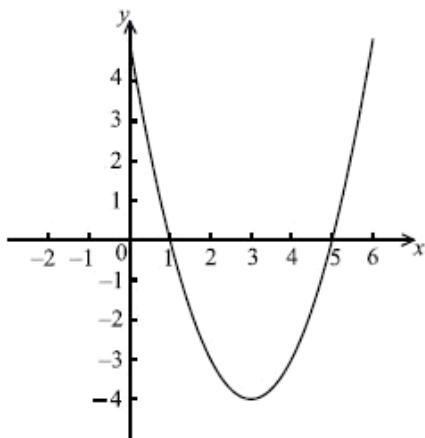
8. The graph of a function f is shown in the diagram below. The point A $(-1, 1)$ is on the graph, and $y = -1$ is a horizontal asymptote.



- (a) Let $g(x) = f(x-1) + 2$. On the diagram, sketch the graph of g .
- (b) Write down the equation of the horizontal asymptote of g .
- (c) Let A' be the point on the graph of g corresponding to point A. Write down the coordinates of A' .

(Total 6 marks)

9. The following diagram shows part of the graph of a quadratic function, with equation in the form $y = (x - p)(x - q)$, where $p, q \in \mathbb{Z}$.



- (a) Write down
- (i) the value of p and of q ;
- (ii) the equation of the axis of symmetry of the curve. (3)
- (b) Find the equation of the function in the form $y = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$. (3)
- (c) Find $\frac{dy}{dx}$. (2)
- (d) Let T be the tangent to the curve at the point $(0, 5)$. Find the equation of T . (2)

(Total 10 marks)

10. Let $f(x) = \log_a x$, $x > 0$.

(a) Write down the value of

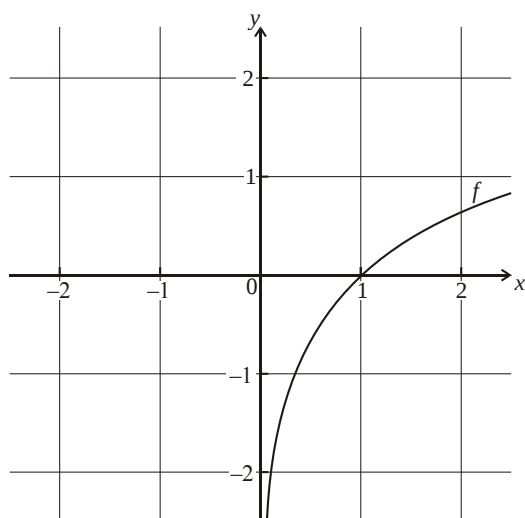
(i) $f(a)$;

(ii) $f(1)$;

(iii) $f(a^4)$.

(3)

(b) The diagram below shows part of the graph of f .



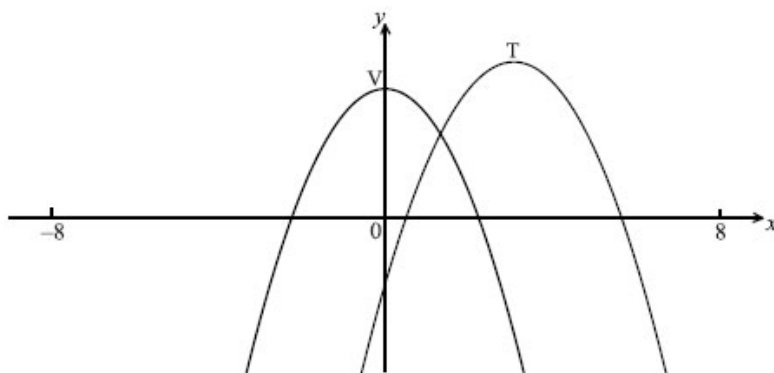
On the same diagram, sketch the graph of f^{-1} .

(3)

(Total 6 marks)

11. The following diagram shows part of the graph of $f(x) = 5 - x^2$ with vertex V (0, 5).

Its image $y = g(x)$ after a translation with vector $\begin{pmatrix} h \\ k \end{pmatrix}$ has vertex T (3, 6).



(a) Write down the value of

(i) h ;

(ii) k .

(2)

(b) Write down an expression for $g(x)$.

(2)

(c) On the same diagram, sketch the graph of $y = g(-x)$.

(2)

(Total 6 marks)