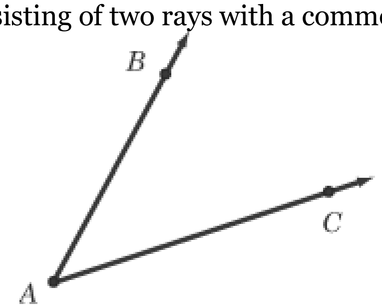
Plane geometry problems account for 14 questions on the ACT Math Test—that's almost a quarter of the questions on the Subject Test. If you've taken high school geometry, you've probably covered all of the topics reviewed here. While you can probably get by on the ACT without knowing trigonometry or intermediate algebra very well, you cannot get by without a solid understanding of plane geometry, because these questions constitute such a significant part of the test.
In this section, we'll cover these plane geometry topics in the following order:

1. Angles and Lines
2. Triangles
3. Polygons
4. Circles
5. Simple Three-Dimensional Geometry

Key topics, such as area and perimeter, will be covered in the relevant sections. For instance, areas of triangles are covered in the section on triangles.

## Angles and Lines

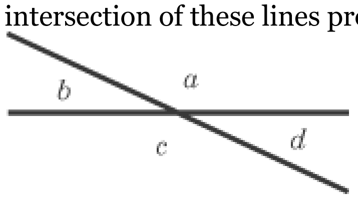
An angle is a geometric figure consisting of two rays with a common endpoint:



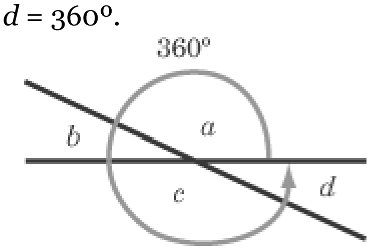

The common endpoint of the rays is called the vertex of the angle. In this case, the vertex is point *A*, which is a part of the ray $\overrightarrow{AB}$ as well as the ray $\overrightarrow{AC}$. The angle can be called either $\angle CAB$ or $\angle BAC$. The only rule in naming an angle is that the vertex *must* always be the middle "initial" of the angle.

### Measuring Angles

Angles are measured in degrees, sometimes denoted by the symbol º. There are 360º in a complete rotation around a point; a circle therefore has 360º.
Consider two intersecting lines. The intersection of these lines produces four angles:




From the diagram below, you should see that the four angles together encompass one full revolution around the two lines' point of intersection. Therefore, the four angles produced by two intersection lines total 360º; angles $a + b + c + d = 360º$.

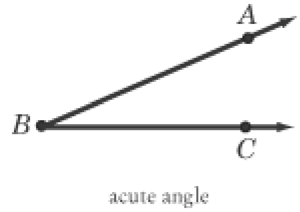


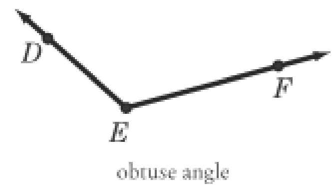
## Types of Angles

There are many different types of angles, all categorized by the number of degrees they have.

### ACUTE AND OBTUSE ANGLES

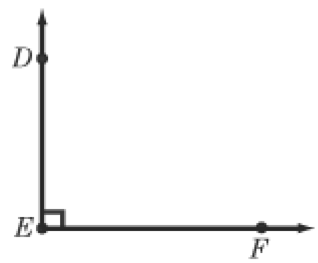
As shown in the diagram below, an acute angle is an angle that is smaller than 90°, while an obtuse angle is an angle that is greater than 90° but less than 180°.



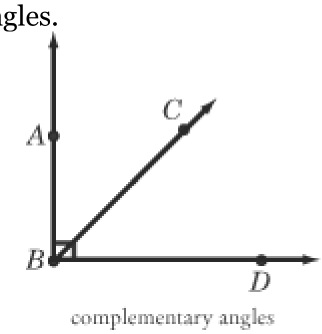

### RIGHT ANGLES

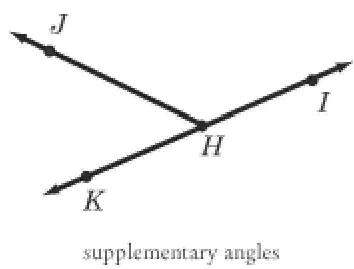
An angle with a measure of 90° is called a right angle. Notice that a right angle is symbolized by a square drawn in the corner of the angle. Whenever you see that little square, you know that you are dealing with a right angle. You also know that the lines that meet at the right angle are perpendicular.




### COMPLEMENTARY AND SUPPLEMENTARY ANGLES

Special names are given to pairs of angles whose sums equal either 90° or 180°. Two angles whose sum is 90° are called complementary angles. If two angles add up to 180°, they are called supplementary angles.

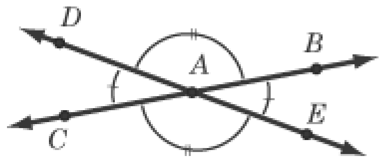



Angles *ABC* and *CBD* are complementary, whereas angles *KHJ* and *JHI* are supplementary.
It is important to remember that these terms are only relative. An angle is only supplementary or complementary to *another specific angle*. A single angle, when considered alone, can be neither supplementary nor complementary—it can only take on one of these properties when considered as part of a pair of angles.

### VERTICAL ANGLES

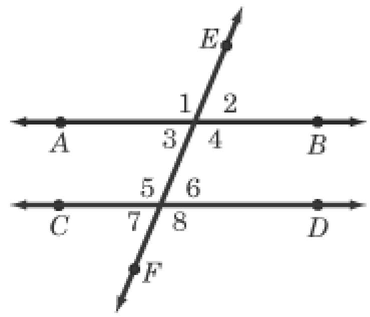
When two lines (or segments) intersect, the angles that lie opposite each other, called vertical angles, are always equal.




Angles *DAC* and *BAE* are vertical angles and are therefore equal to each other. Angles *DAB* and *CAE* are also vertical (and equal) angles.

### Parallel Lines Cut by a Transversal

Occasionally on the ACT, you will run into a problem in which two parallel lines are cut by a third straight line, known as a transversal. The eight angles created by these two intersections have special relationships with one another.




Angles 1, 4, 5, and 8 are all equal to each other. So are angles 2, 3, 6, and 7. Also, the sum of any two adjacent angles, such as 1 and 2 or 7 and 8, equals 180°. From these rules, you can make justified claims about seemingly unrelated angles. For example, since angles 1 and 2 add up to 180°, and since angles 2 and 7 are equal, the sum of angles 1 and 7 also equals 180°, based on the substitution principle of addition.

### Lines

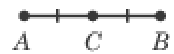
You may see a problem on the ACT that asks you about lines. In order to understand these questions, there is some vocabulary that you need to know.

- **Line.** A line is a set of infinite points that runs straight. If you have two points, the line will run straight through them and extend infinitely in both directions.

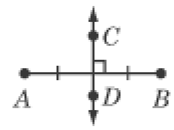

- **Line Segment.** A line segment consists of two points (endpoints) and all the points on a straight line between them. If you have a line segment that stretches from point *A* to point *B*, the line segment will be referred to as $\overline{AB}$ .


- **Ray.** A ray is a line that has one endpoint; it extends infinitely in the direction without the endpoint.

- **Midpoint.** A midpoint is the point exactly halfway between the two endpoints of a line segment.




- **Bisect (verb).** Anything that bisects a line segment cuts the line segment exactly in half, at the midpoint. Line $\overleftrightarrow{CD}$ bisects line segment $\overline{AB}$.



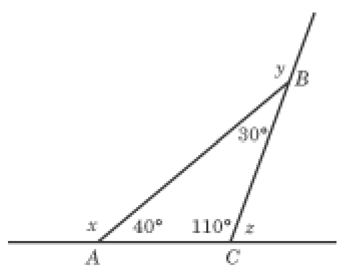

And that's all the line vocab you need for the ACT Math Test.

## Triangles

On each ACT Math Test, you will see three or four questions on triangles. These questions tend to deal with the angles and sides of triangles, but you may also see questions about their areas and perimeters.

Triangles are closed figures containing three angles and three sides. There are a number of important rules about these angles and sides which, if mastered, will take you a long way on the ACT.

- The sum of the three angles in a triangle will always equal 180°. Thus, if you know the measure of two angles in a triangle, you can calculate the measure of the third angle.
- The exterior angle of a triangle is always equal to the sum of the remote interior angles (i.e., the angles that are not adjacent to the exterior angle). In the figure, the exterior angle, *x*, is equal to 140°, which is the sum of the two remote interior angles.

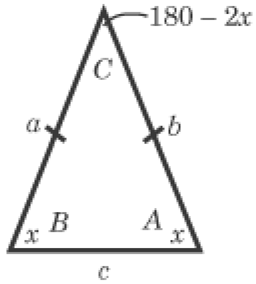



- The sum of the exterior angles of a triangle will always equal 360°, therefore $\angle x + \angle y + \angle z = 360°$.
- The longest side of a triangle is always opposite the largest angle; the second-longest side is opposite the second-largest angle; the shortest side is opposite the smallest angle. Therefore, in the triangle above, $\angle ACB$ is the largest angle, because its opposite side $\overline{AB}$, is the longest side.
- No side of a triangle can be as long as the sum of the other two side lengths. Therefore, in the triangle above, $\overline{AB} < \overline{BC} + \overline{CA}$.
- If you know that a triangle has sides of length 4 and 6, you know the third side is shorter than 10 and longer than 2. This can help you eliminate possible answer choices on multiple-choice questions.

There are a number of specialized types of triangles. We'll discuss them below.

## Isosceles Triangles

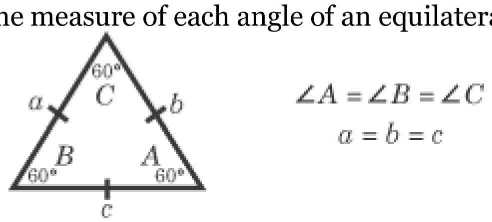
Isosceles triangles have two equal sides, in this case sides *a* and *b* (the little marks on those two sides mean that the sides are congruent, which means equal). The angles opposite the congruent sides, in this case angles *A* and *B*, are also equal.




Because these two angles are equal, and the sum of a triangle's angles is always 180°, if you know the value of one of the two equal angles, let's say angle *A*, you know the value of all the angles in the triangle. Angle *B* is equal to *A*. Angle *C* is equal to 180 – 2*A* (since *A* and *B* are equal, *A* + *B* = 2*A*). The same is true if you start with the measure of angle *C*: angles *A* and *B* each measure (180 – *C*) / 2.

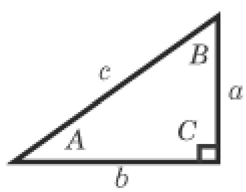
## Equilateral Triangles

An equilateral triangle is a triangle in which all the sides and all the angles are equal. Since the angles of a triangle must total 180°, the measure of each angle of an equilateral triangle must be 60°.




## Right Triangles

A triangle with a right angle (90°) is called a right triangle. Because the angles of a triangle must total 180°, the nonright angles (angles *A* and *B* in the diagram below) in a right triangle must add up to 90° (that is, they are complementary). The side opposite the right angle (side *c* in the diagram below) is called the hypotenuse.




## The Pythagorean Theorem

The Pythagorean theorem defines the relationship between the sides of every right triangle. The theorem states that the length of the hypotenuse squared is equal to the sum of the squares of the lengths of the legs:

$$c^2 = a^2 + b^2$$

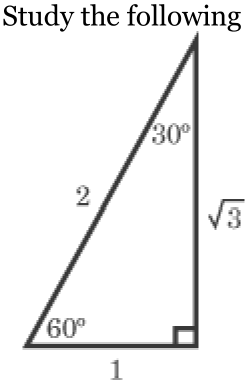
If you are given any two sides of a right triangle, you can use the Pythagorean theorem to calculate the length of the third side.

Certain groups of three integers can be the lengths of a right triangle. Such groups of integers are called Pythagorean triples. Some common Pythagorean triples include {3, 4, 5}, {5, 12, 13}, {8, 15, 17}, {7, 24, 25}, and {9, 40, 41}. Any multiple of one of these groups is also a Pythagorean triple. For example, {9, 12, 15} = 3{3, 4, 5}. If you know these basic Pythagorean triples, they can help you quickly determine, without calculation, the length of a side of a right triangle in a problem that gives you the length of the other two sides.

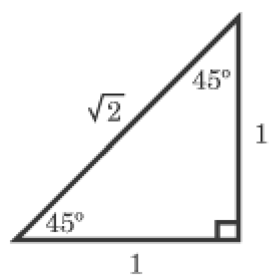
## Special Right Triangles

There are two kinds of special right triangles for which you don't have to use the Pythagorean theorem because their sides always exist in the same distinct ratios. This is not to say that you *can't* use the Pythagorean theorem when dealing with these triangles, just that you don't have to, since you can work out problems very quickly if you know the ratios. The two types of triangles are called 30-60-90 and 45-45-90 right triangles.

A 30-60-90 triangle is, as you may have guessed, a triangle with angles of 30°, 60°, and 90°. What makes it special is the specific pattern that the side-length of 30-60-90 triangles follow. Suppose the short leg, opposite the 30° angle, has length *x*. Then the hypotenuse has length 2*x*, and the long leg, opposite the 60° angle, has length $x\sqrt{3}$. Study the following diagram, which shows these ratios:

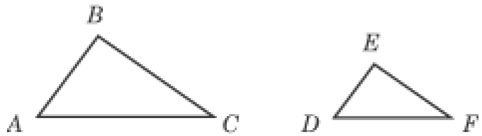



A 45-45-90 triangle is a triangle with two angles of 45° and one right angle. This type of triangle is also known as an isosceles right triangle, since it's both isosceles and right. Like the 30-60-90 triangle, the lengths of the sides of a 45-45-90 triangle also follow a specific pattern that you should know. If the legs are of length *x* (they are always equal), then the hypotenuse has length $x\sqrt{2}$. Take a look at this diagram:




## Similarity in Triangles

In reference to triangles, the word similar means "shaped in the same way." Two triangles are similar if their corresponding angles are equal. If this is the case, then the lengths of corresponding sides will be proportional to each other. For example, if triangles *ABC* and *DEF* are similar, then sides $\overline{AB}$ and $\overline{DE}$ correspond to each other, as do $\overline{BC}$ and $\overline{EF}$, and $\overline{CA}$ and $\overline{FD}$.

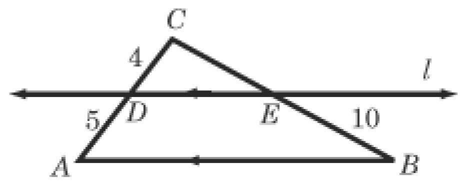



The proportionality of corresponding sides means that:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

The properties of similarity will almost definitely be tested on the ACT. Let's say you come across the following question:

Triangles *ABC* and *DEC* are similar, and line *l* is parallel to segment $\overline{AB}$. What is the length of $\overline{CE}$?

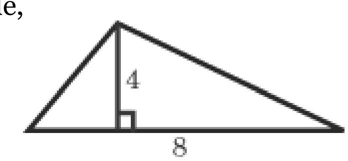



If you know the rule of similarity, you will quickly realize that the ratio $\overline{CD}:\overline{CA}$ is 4:9 and that $\overline{CE}:\overline{CB}$ must have the same ratio. Since $\overline{EB}$ is equal to 10, the only possible length of $\overline{CE}$ is 8, since 8:18 is equivalent to 4:9.

Two triangles are similar if they have two pairs of corresponding angles and one pair of sides that are equal, or if one pair of angles is equal and the two pairs of adjacent sides are proportional.

### Area of a Triangle

The area of a triangle is equal to one-half the base of the triangle times the height, or $(1/2)bh$. For example, given the following triangle,




the area equals $(1/2)(4 \times 8) = 16$. If you know the length of one leg of a triangle and can determine the height of the triangle (using that leg as a base), then you can plug those two numbers into the area formula.

### Perimeter of a Triangle

The perimeter of a triangle is equal to the sum of the lengths of the triangle's three sides. If a triangle has sides of length 4, 6, and 9, then its perimeter is 4 + 6 + 9 = 19.

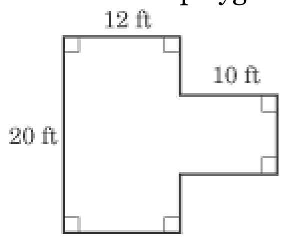
## Polygons

Polygon questions on the Math Test tend to deal with perimeters and areas, so you should pay particular attention to those sections below.

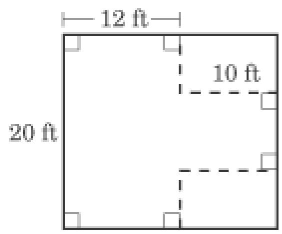
By definition, a polygon is a two-dimensional figure with three or more straight sides. Under that definition, triangles are a type of polygon. However, since triangles are such an important part of the ACT, we gave them their own section. This section will deal with polygons of four sides or more.

## Perimeter of Polygons

As with triangles, the perimeter of a polygon is equal to the sum of the length of its sides. Perimeter problems on the ACT usually involve unconventional polygons like this one:



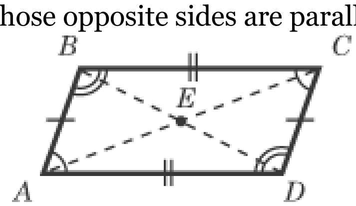

If you saw this polygon on the ACT, you would probably be asked to determine its perimeter. These questions can be tricky because of the number of sides involved and the number of sides the ACT decides to label. Wouldn't it have been easier if all the sides were labeled? Yes, it would have been easier, which is why the ACT didn't label them. Still, if you think about it, this question isn't hard. In fact, if you flipped the lines out in the upper-right and lower-right corners, you would have a rectangle:




The ACT writers were kind enough to give you the height of the rectangle (20 ft), and you can figure out the width of the rectangle by adding 12 ft and 10 ft to get 22 ft. So the perimeter of this normal rectangle masquerading as a weird polygon is $2 \times l + 2w$ or $2 \times 20 + 2 \times 22 = 84$ ft.

## Parallelograms

A parallelogram is a quadrilateral whose opposite sides are parallel.

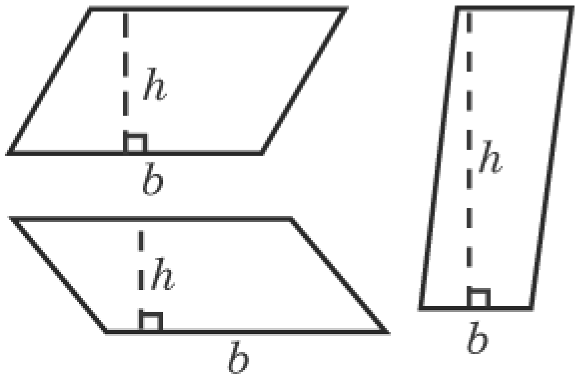



In a parallelogram,

1. Opposite sides are equal in length; $\overline{BC} = \overline{AD}$ and $\overline{BA} = \overline{CD}$.
2. Opposite angles are equal; $\angle A = \angle C$; $\angle B = \angle D$.
3. Adjacent angles are supplementary.
4. The diagonals bisect each other; therefore, $\overline{BE} = \overline{ED}$ and $\overline{AE} = \overline{EC}$.
5. Each diagonal splits a parallelogram into two congruent triangles; $\Delta AEB \cong \Delta CED$.
6. Two diagonals split a parallelogram into two pairs of congruent triangles.

### AREA OF A PARALLELOGRAM

To calculate the area of a parallelogram, we must introduce a new term: altitude. The altitude of a parallelogram is the line segment perpendicular to a pair of opposite sides with one endpoint on each side. Below are various parallelograms and their altitudes.

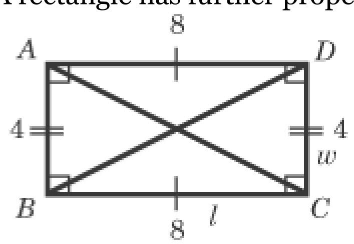



The area of a parallelogram is the product of the length of its altitude and the length of a side that contains an endpoint of the altitude. This side is called the base of the parallelogram. Any side can become a base of a given parallelogram: all you need to do is draw an altitude from it to the opposite side. A common way to describe the area of a parallelogram is the base times the height, where the height is the altitude:

$$A = b \times h$$

## Rectangles

A rectangle is a specialized parallelogram whose angles all equal 90°. All the rules that hold for parallelograms hold for rectangles. A rectangle has further properties, however:




In a rectangle,

1. The angles are all equal to 90°.
2. The diagonals are equal in length; $\overline{BD} = \overline{AC}$.

### AREA OF A RECTANGLE

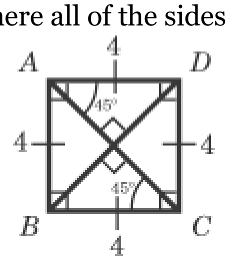
The area of a rectangle is equal to its length multiplied by its width:

$$A = lw$$

In the case of the rectangle pictured above, the area equals $4 \times 8 = 32$ square units.

## Squares

A square is a specific kind of rectangle where all of the sides are of equal length.



In a square,

1. All sides are of equal length.
2. All angles are equal to 90°.
3. The diagonals bisect each other at right angles; $\overline{BD} \perp \overline{AC}$.
4. The diagonals bisect the vertex angles to create 45° angles. (This means that the two diagonals break the square into four 45-45-90 triangles.)
5. The diagonals are equal in length; $\overline{BD} = \overline{AC}$.

#### AREA OF A SQUARE

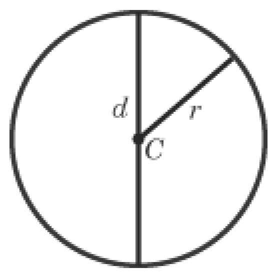
The area of a square is equal to the square of the length of a side:

$$A = s^2$$

In the case of the square above, the area is 16. Notice that the calculation for a square's area is essentially the same calculation as that of a rectangle's area (length times width).

## Circles

You may encounter one or two circle questions on the Math Test. As we said in the coordinate geometry section, a circle is the set of all points equidistant from a given point. The point from which all the points on a circle are equidistant is called the center, and the distance from that point to the circle is called the radius.




The circle above has its center at point C and a radius of length *r*. All circles also have a diameter. The diameter of a circle is a line segment that contains the center and whose endpoints are both on the circle. The length of the diameter is twice that of the radius.

### Circumference of a Circle

The formula to find the circumference of a circle is:

$$C = 2\pi r$$

where *r* stands for the length of the radius. Because two times the radius is also equal to a circle's diameter, the formula for the circumference of a circle can also be written as $\pi d$.

### Area of a Circle

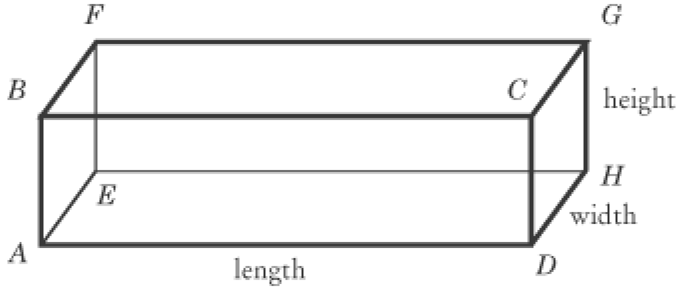
The area of a circle is the radius squared multiplied by $\pi$:

$$A = \pi r^2$$

## Simple Three-Dimensional Geometry

Solids are three-dimensional shapes. You probably will not see any questions on solids on the Math Test. When these questions do show up, they almost always cover rectangular solids, which are the easiest solids with which to work.

A rectangular solid is a six-faced, three-dimensional shape with six rectangular faces.




Just as squares are specialized rectangles, cubes are specialized rectangular solids. For a cube, the length, width, and height are all equal.

### Surface Area

The surface area of a solid is the area of its outermost skin. A cardboard box, for example, is made up of a bunch of rectangles fastened together. The sum of the areas of those rectangles is the surface area of the cardboard box.

To calculate the surface area of a rectangular solid, all you have to do is find the area of each of the sides and sum them. In fact, your job is even easier than that. The six sides of a rectangular solid can be divided into three pairs of two. If you look at the solid diagrammed above, you should see that panel *ABFE = DCGH, BCDA = FGHE*, and *BCGF = ADHE*. Therefore, you only have to calculate the areas of one of each of the three pairs, sum those areas, and multiply that answer by two.

With a cube, finding the surface area is even easier. By definition, each side of a cube will always be the same, so to calculate the surface area, find the area of one side and multiply by six.

There is one property of surface area of which you should be aware. Imagine a rectangular solid that has a length of 8, a width of 4, and a height of 4. Now imagine a giant cleaver that comes and cuts the solid into two cubes, each of which has a length, width, and height of 4. Do the two cubes have a bigger combined surface area, a smaller combined surface area, or a combined surface area equal to the original solid? The answer is that the two cubes have a bigger surface area. Think about the cleaver coming down: it cuts the original solid in half, meaning it creates two new faces that are now on the surface. Whenever something is cut in half, or in pieces, its surface area increases (although its volume is unchanged).

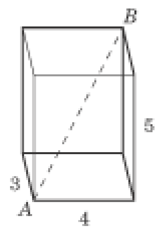
### Volume

The volume of a rectangular solid can be found by multiplying the length by the width by the height. In other words, $V = lwh$.

Because all the dimensions of a cube are equal, the volume of a cube is even easier to calculate: just raise the length of one edge to the third power. If a cube has a length, width, and height of 3, the volume equals $3^3 = 27.$

### Diagonal Length

The diagonal of a rectangular solid is the line segment whose endpoints are at opposite corners. Each rectangular solid has four diagonals, all with the same length, which connect each pair of opposite vertices.




The formula for the length of a diagonal is:

$$d = \sqrt{l^2 + w^2 + h^2}$$

where *l* is the length, *w* is the width, and *h* is the height.
You can think of this formula as the Pythagorean theorem in three dimensions. In fact, you can derive this formula using the Pythagorean theorem. First, find the length of the diagonal along the base. This is $\sqrt{l^2 + w^2}$. Then use the Pythagorean theorem again, incorporating height to find the length of the diagonal from one corner to the other: $d^2 = (\sqrt{l^2 + w^2})^2 + h^2$. Thus $d^2 = l^2 + w^2 + h^2$ and $d = \sqrt{l^2 + w^2 + h^2}$.

http://www.sparknotes.com/testprep/books/act/chapter2.rhtml