Elementary Algebra

The elementary algebra section is organized in terms of increasing difficulty:

1. Substitution
2. Simplifying Algebraic Expressions
3. Writing Expressions and Equations
4. Solving Linear Equations
5. Multiplying Binomials
6. Inequalities

This is also the order of the topics in terms of *decreasing frequency* on the test, with one exception: problems involving inequalities pop up more often than problems involving binomial multiplication. Before covering these topics, however, we will address a question brought up by the teachings of some other test prep companies.

To Algebra or Not to Algebra

There are many ways to answer most algebra problems. You can use algebra—setting up and working out equations—or you can plug numbers into equations to try and avoid using algebra. In some cases, you might even be able to solve a question by being a particularly intuitive genius and finding a magnificent shortcut.

We want to stress that none of these methods is necessarily better than another. Which method is best for you depends on your math ability and your target score. Trying to solve problems with algebra is more conceptually demanding, but can take less time. Plugging in numbers makes questions easier to understand, but will likely take more time. In general, if you are uncomfortable with algebra, you should try to use the plugging-in method. If you are comfortable with algebra, using it is probably the best way to go. Still, these suggestions are not carved in stone. If you are generally comfortable with algebra but come upon a question that is stumping you, try plugging in answers. If you usually prefer plugging in answers but come upon a question you can answer using algebra, use algebra. When you study your practice tests and look at the algebra questions you got wrong, you should think about the method you employed. Did you plug in when you should have used algebra? Did you use algebra when you should have plugged in? As for being an intuitive math genius, it just can't be taught—though we will show you how one might think.
Here's a sample algebra question:

A man flipped a coin 162 times. The coin landed with the heads side up 62 more times than it landed with tails up. How many times did the coin land heads?

A. 100
B. 104
C. 108
D. 112
E. 116

Solving by Plugging In

If you were to answer this problem by plugging in numbers, you would pick the middle number or C, 108, as the first number to try, since if it does not happen to be the answer, you can discard the numbers smaller than it or larger than it. If the coin came up heads 108 times, then how many times did it land tails? It landed tails 162 – 108 = 54 times. Is 108 heads landings 62 more than 54 tails landings? No, 108 – 54 = 54. In order for the problem to work out, you need more heads landings. You can eliminate A and B as possibilities. Let's say we choose D, 112, as our next plug-in number: 162 – 112 = 50. Does 112 – 50 = 62? Yes. **D** is the answer.

Solving with Algebra

If you answer this question with algebra, you realize that if heads are represented by the variable x, then tails are represented by $(x – 62)$. Therefore,

$$\begin{aligned} x + (x - 62) &= 162 \\ 2x - 62 &= 162 \\ 2x &= 224 \\ x &= 112 \end{aligned}$$

As you can see, there's simply less math to do for this problem when you use algebra. Using algebra will only take you longer than plugging in if you have trouble coming up with the equation $x + (x – 62) = 162$.

Therefore, if you can quickly come up with the necessary equation, then use algebra to solve algebra problems. If you have the feeling that it will take you a while to figure out the correct equation, then plug in.

Solving by Being an Amazing Genius

It is quite possible that you just looked at this problem and said to yourself, "Other than the 62 more heads, all the other flips were equally heads and tails. So if you take the 62 out of the total of 162, then you know that the other 100 flips were 50 heads and 50 tails. Now I can just add 62 + 50 = 112. Man, I am an amazing genius!"

The Bottom Line on Using Algebra

Hopefully, our example has convinced you that there isn't any "right way" to answer a question dealing with algebra. There are faster ways and slower ways, and it always benefits you to use the faster way if you can, but the most important thing is getting the question right. Therefore, when you come to a question, don't insist on using only one method to try to answer it. Just do what you have to do in order to answer the question correctly in as little time as possible.

Now we'll begin to cover the topics of elementary algebra tested on the ACT.

Substitution

Substitution questions are the simplest algebra problems on the ACT. These questions provide you with an algebraic expression and the value of a variable within the equation, and ask you to calculate the value of the equation. For example,

If $2y + 8x = 11$, what is the value of $3(2y + 8x)$?

You might see this question with all its variables and panic. But, in truth, this is a simple problem. Since $2y + 8x = 11$, all you have to do is substitute 11 for $2y + 8x$ in the expression $3(2y + 8x)$, and you get $3(11) = 33$.

For some substitution questions, you will have to do some simple math either before or after the substitution.

Math before Substitution

If $3x - 7 = 8$, then $23 - 3x =$

In this problem you have to find what $3x$ equals before you can substitute that value into the expression $23 - 3x$. To find $3x$, take:

$$3x - 7 = 8$$

and add 7 to both sides, getting:

$$3x = 15$$

Now we can substitute that 15 into $23 - 3x$:

$$23 - 15 = 8$$

Math after Substitution

If $a + b = 7$ and $b = 3$, then $4a =$

Here we first have to solve for a by substituting the 3 in for b:

$$a + b = 7$$
$$a + 3 = 7$$
$$a = 4$$

Once you know that $a = 4$, just substitute it into $4a$:

$$4 \times 4 = 16$$

Simplifying Algebraic Expressions

Some ACT Math questions test your ability to simplify or manipulate algebraic expressions. To master either of these skills, you must be able to see how an equation might be expressed differently without changing the value of the expression in any way. There are two primary ways to simplify an equation: factoring and combining like terms.

Factoring and Unfactoring

Factoring an algebraic expression means finding factors common to all terms in an expression and dividing them out. For example, to factor $3a + 3b$, you simply divide out the 3 to get $3(a + b)$. Below are some more examples of factoring:

1. $6y + 8x = 2(3y + 4x)$
2. $8b + 24 = 8(b + 3)$
3. $3(x + y) + 4(x + y) = (3 + 4)(x + y) = 7(x + y)$
4. $\frac{2x + y}{x} = \frac{2x}{x} + \frac{y}{x} = 2 + \frac{y}{x}$

Unfactoring involves taking a factored expression, such as $8(b + 3)$, and distributing one term to the other(s): $8b + 24$.

Combining Similar Terms

If an expression contains "like terms," you can combine those terms and simplify the equation. "Like terms" refers to identical variables that have the same exponent value. For example:

- You can combine: $x^2 + 8x^2 = 9x^2$ $y^{13} + 754y^{13} = 755y^{13}$ $m^3 + m^3 = 2m^3$

As long as two terms have the same variable and the same exponent value, you can combine them. Note that when you combine like terms, the variable doesn't change. If two terms have different variables, or the exponent value is different, the terms are not "like terms" and you cannot combine them.

- You can't combine: $x^4 + x^2$ or $y^2 + x^2$

Writing Expressions and Equations

Occasionally, the ACT will throw you a word problem that describes an algebraic expression. You will have to write out the expression in numerical form and perhaps simplify it. For example:

Mary poured *g* cups of water into a bucket, leaving the bucket with a total of *f* cups in it. Mary then removed (*g* – 3) cups of water from the bucket. How many cups of water remain in the bucket?

To answer this question, you have to interpret the word problem. In other words, you have to figure out what is important in the word problem and how it fits into the expression you need to build. This question asks you to generate an expression that describes how many cups of water there are in the bucket *after* Mary removes (*g* – 3) cups. It doesn't matter what *g* actually equals, because we don't care how much water was in the bucket before Mary added *g* cups.
To work out the equation, we take the original number of cups in the bucket and subtract from it what was removed:

$$f - (g - 3) = f - g + 3$$

Solving Linear Equations

The most common and foolproof way to solve linear equations is to isolate the variable whose value you are trying to determine on one side of the equation.
If you stay alert, you may also be able to find shortcuts that will greatly reduce your time spent per question without affecting your accuracy. Let's look at an easy example:

If $6p + 2 = 20$, then $6p - 3 =$

This is an easy problem to solve through the normal algebraic method. First we solve for *p*:

$$6p + 2 = 20$$
$$6p = 18$$
$$p = 3$$

Next, we plug 3 into the second equation:

$$6p - 3 =$$
$$6(3) - 3 =$$
$$18 - 3 = 15$$

But there's a faster way to answer this question. The secret is that you don't have to solve for p at all. Instead, notice that both equations contain $6p$ and that the value of $6p$ will not change. Therefore, all you have to do in the first equation is solve for $6p$. And as you can see above, that simply means subtracting 2 from 20 to get 18. Once you know $6p$ is 18, you can plug 18 in for $6p$ in the second equation and get your answer.

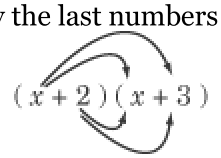
When you come upon an algebra question asking you to solve an equation, you should always take a second to look for shortcuts. Look for equations that not only have the same variables, but also the same coefficients attached to that variable (such as $6p$ and $6p$). If you are able to find a good shortcut, your knowledge of algebra will save you time.

Multiplying Binomials

A binomial is an algebraic expression consisting of two terms combined by a plus or minus sign. For instance, $x + 4$ and $y - 11$ are both binomials. Multiplying binomials is not a difficult task if you remember the acronym FOIL, which stands for FIRST OUTER INNER LAST. For example, say you are asked to multiply the binomials:

$$(x+2)(x+3)$$

You start by multiplying the first number in each polynomial $(x)(x)$, then the outer numbers $(x)(3)$, then the inner numbers $(2)(x)$, and finally the last numbers $(2)(3)$:



and you get:

$$x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

The only tricky part to following FOIL is remembering to pay attention to signs. For instance, if you have the polynomials $(x + 2)(x - 3)$, then the -3 comes to play an important part. You always add up the products of FOIL, but look what happens when there's a negative number involved:

$$(x+2)(x-3) = x^2 + 2x + (-3x) + (-6) = x^2 + 2x - 3x - 6$$
$$= x^2 - x - 6$$

There are three equations involving binomial multiplication that you should know backward and forward before you take the ACT, the most important being the first:

$$(x+y)(x-y) = x^2 - y^2$$
$$(x+y)(x+y) = x^2 + 2xy + y^2$$
$$(x-y)(x-y) = x^2 - 2xy + y^2$$

Inequalities

An equation states that the quantities on either side of the equal sign are of the same value. An inequality states that one side of the equation is greater than the other: $a < b$ states that a is less than b, and $a > b$ states that a is greater than b. In other cases, $a \leq b$ means that a is less than or equal to b, while $a \geq b$ means that a is greater than or equal to b.

Solving an inequality is basically the same as solving a normal equation: all the rules of simplification and having to do the same thing to both sides still apply. The one rule that does differ when working with inequalities comes when you multiply or divide both sides by a negative. If you do so, you must flip the sign: if $x > y$, then $-x < -y$. For example, if you have $2x + 6 \geq y$ and multiply the inequality by −2, the result is $-4x - 12 \leq -2y$.

Intermediate Algebra

Intermediate algebra questions are some of the toughest questions on the ACT Math Test. To compensate for the difficulty of the topic, almost all of the intermediate algebra problems will be in basic form, meaning that you don't need to sort through a mess of words to find the question. Also, you should be glad to hear that there will be only nine intermediate algebra problems on the Math Test, making them worth less than one-sixth of your math score.
In this section, we'll present the intermediate algebra topics to you in the following order:

1. 1. Solving and Factoring Quadratic Equations
2. Solving Systems of Equations
3. Relationships between the Sides of an Equation
4. Functions
5. Matrices
6. Logarithms

The first two topics in this list appear most frequently on the ACT. You may not encounter a single example of the last four topics, particularly the last two, on a given test. Those topics do appear from time to time, though, so it pays to be prepared for them.

Solving and Factoring Quadratic Equations

This topic constitutes a major portion of the intermediate algebra questions. You will probably see about three quadratic equations questions per test. Those three questions make up a third of the intermediate algebra questions. If you can master these questions, then you're well on your way to overcoming intermediate algebra.

Definition of a Quadratic Equation

A quadratic equation is a second-degree equation with one variable and usually two solutions. If you don't understand what that means, hold on a second. A quadratic equation on the ACT will almost always appear in the following form:

$$ax^2 + bx + c = 0$$

where *a* and *b* are coefficients and $a \neq 0$. That is the standard form of a quadratic equation, and it is the form that the ACT almost always uses. In some cases, you may come across an equation that looks like this:

$$ax^2 + c = bx$$

In this case you can subtract *bx* from both sides of the equation to get the equation into standard form:

$$ax^2 - bx + c = 0$$

Every quadratic equation contains a variable raised to the second power. In most of the quadratic equations you'll see on the test, there'll be two solutions for this variable.
Also, the ACT almost always makes *a* equal to 1 to simplify solving these equations. (Note that when $a = 1$, $1x^2$ is simply written x^2.)

Solving a Quadratic Equation

Solving a quadratic equation means solving for the variable used in the equation. Almost all quadratic equations appearing on the ACT can be solved by factoring. Solving a quadratic equation by factoring is essentially the reverse of what you do when multiplying binomials. Take the following example:

$$x^2 + 9x + 18 = 0$$

Solving for *x* here requires a good degree of intuition, but with time and practice your intuition will become increasingly keen. Try to imagine which binomials would create the equation above. You can do this by considering the factors of 18 (1 and 18; 2 and 9; 3 and 6) and asking yourself which pair of factors adds up to 9. Done that? If so, you see that 3 and 6 add up to 9. So you can factor the equation as:

$$(x + 3)(x + 6) = 0$$

Whenever you see something in the above form, you can solve it like this:

$$\begin{array}{ll} x + 3 = 0 & x + 6 = 0 \\ x = -3 & x = -6 \end{array}$$

Either $x = -3$ or $x = -6$ satisfies the equation.

THE QUADRATIC FORMULA

Very rarely on the ACT, you may encounter a quadratic equation that cannot be solved by factoring. In that case, you can use the quadratic formula to solve the equation. The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where *a*, *b*, and *c* are the same coefficients as in the quadratic equation. You simply plug in the coefficients to determine solutions for *x*. Just to prove to you that the equation works, we'll work out the quadratic equation whose roots we already know: $x^2 + 9x + 18 = 0$. Remember that it's in the form of $ax^2 + bx + c = 0$.

$$x = \frac{-9 \pm \sqrt{9^2 - 4 \times 1 \times 18}}{2 \times 1}$$

$$x = \frac{-9 \pm \sqrt{81 - 72}}{2}$$

$$x = \frac{-9 \pm \sqrt{9}}{2}$$

$$x = \frac{-9 \pm 3}{2}$$

$$x = \frac{-6}{2} = -3 \text{ or } x = \frac{-12}{2} = -6$$

Solving Systems of Equations

A few times per test, the ACT will give you two equations and ask you to determine the value of a particular variable or some other equation or expression. For example,

If $3x + 4y = 32$ and $2y - x = 6$, then $x - y =$

The best way to answer this type of question is to use a substitution method: solve for one variable and then substitute that value into the other equation. In looking at the two equations above, it seems obvious that it would be easier to solve for x using the second equation than it would be to solve for y in either of the two equations. All it takes is a little reorganizing:

$$2y - x = 6$$
$$2y - 6 = x$$
$$x = 2y - 6$$

Next, all we have to do is plug $2y - 6$ into the value for x in the first equation:

$$3(2y - 6) + 4y = 32$$

Now we have only one variable to deal with in the equation, and we can easily solve for it:

$$6y - 18 + 4y = 32$$
$$10y = 50$$
$$y = 5$$

Once we know the value of y, we can plug that value into either equation to solve for x:

$$x = 2y - 6$$
$$x = 2(5) - 6$$
$$x = 4$$

Now we can answer the question, which asked for $x - y$. We get $4 - 5 = -1$. When you solve systems of equations questions, always be careful of a few things:

1. When you first solve for one variable, make sure you solve for it in its lowest form (solve for x rather than $2x$).
2. When you substitute, make sure you correctly apply the distributive law of multiplication: $3(2y - 6) = 6y - 18$.
3. Always answer the question the ACT asks. For example, in the sample above, the question asked for the value of $x - y$. But it's certainly possible that after doing all the work and figuring out that $x = 4$ you might forget to carry out the final simple operation of $4 - 5 = -1$, and instead incorrectly answer 4.

Systems of Equations with Infinite Solutions

Occasionally, the ACT will test your understanding of systems of linear equations by asking you to determine when two equations in a system yield an infinite number of solutions. To answer this sort of question, you only need to know one thing: a system of equations will yield an infinite number of solutions when the two equations describe the same line. In other words, the system of equations will have an infinite number of solutions when the two equations are equal and in $y = mx + b$ form. Here's an example:

The following system of equations would have an infinite number of solutions for which of the following values of b?

$3x - 2y = 4$

$12x - 4by = 16$

A. 1
B. 2
C. 4
D. 8
E. 12

To answer this question, you have to pick a value for b such that the two equations have the same formula fitting the $y = mx + b$ form. The first step in this process is to transfer $3x - 2y = 4$ into the $y = mx + b$ form:

$$\begin{aligned} 3x - 2y &= 4 \\ -2y &= -3x + 4 \\ y &= \frac{3}{2}x - 2 \end{aligned}$$

Then put $12x - 4by = 16$ into the same form:

$$\begin{aligned} 12x - 4by &= 16 \\ -4by &= -12x + 16 \\ by &= 3x - 4 \\ y &= \frac{3x}{b} - \frac{4}{b} \end{aligned}$$

Since you know that the two equations have to be equal, you know that $(3/2)x$ must equal $3x/b$. This means that $b = 2$, so **B** is the right answer to the question.

Relationships between the Sides of an Equation

You should understand the relationship between the two sides of an equation. If you have an equation that says $w = kt^2$, where k is a constant, the equation tells you that w varies directly with the square of t; in other words, as t increases, so does w.

If, on the other hand, you have an equation that says $w = k/t^2$, where k is a constant, the equation tells you that w varies inversely with the square of t; in other words, as t increases, w decreases.

Functions

If you restated $y = ax + b$ as $f(x) = ax + b$, you would have a function, $f(x)$, which is pronounced "f of x." On the ACT, you can almost always treat $f(x)$ as you would treat y, but we want you to be aware of the different format.

Compound Functions

On rare occasions, the ACT has asked questions about compound functions, in which one function is worked out in terms of another. The notation for a compound function is $f(g(x))$, or $f \circ g$. To evaluate a compound function like $f(g(x))$, first evaluate g at x. Then evaluate f at the results of $g(x)$. Basically, work with the inner parentheses first, and then the outer ones, just like in any other algebraic expression. Try the following example:

Suppose $h(x) = x^2 + 2x$ and $j(x) = \left|\frac{x}{4} + 2\right|$. What is $j(h(4))$?

$$\begin{aligned}(j \circ h)(4) &= j(h(4)) \\ &= j(4^2 + 2(4)) \\ &= j(16 + 8) \\ &= j(24) \\ &= |\tfrac{24}{4} + 2| \\ &= |8| \\ &= 8\end{aligned}$$

Here's a slightly more complicated example:

Suppose $f(x) = 3x + 1$ and $g(x) = \sqrt{5x}$. What is $g(f(x))$?

This question doesn't ask you to evaluate the compound function for a given value—it asks you to express the compound function as a single function. To do so, simply plug the formula for *f* into the formula for *g*:

$$\begin{aligned}g(f(x)) &= g(3x + 1) \\ g(f(x)) &= \sqrt{5(3x + 1)} \\ g(f(x)) &= \sqrt{15x + 5}\end{aligned}$$

Matrices

You will seldom see a matrix problem on the ACT, and many high school math courses may not have covered matrices by the time you take the test. Still, any matrix problems on the ACT will be very straightforward and fundamental, so you really only need to know the basics of matrices in order to get the right answers. We will cover those basics here.

Adding and Subtracting Matrices

You will not be asked to do anything more advanced than adding or subtracting matrices. For example,

$$A = \begin{bmatrix} 2 & 0 \\ 3 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & 1 \\ 6 & 3 \end{bmatrix}$$

What is $A + B$? To answer this question, you simply add the corresponding entries in A and B. The entries in the first row are 2 + (–4) = –2 and 0 + 1 = 1. The entries in the second row are 3 + 6 = 9 and (–5) + 3 = –2. So the resulting matrix is:

$$A + B = \begin{bmatrix} -2 & 1 \\ 9 & -2 \end{bmatrix}$$

If the question had asked you what $A – B$ is, then you would simply subtract the entries in B from the corresponding entries in A.

Logarithms

Like matrices, logarithms rarely appear in the ACT Math Test. But they do pop up occasionally, and you should know how to handle them. Logarithmic functions are inverses of exponential functions. The exponential equation $x = a^b$ is equivalent to the logarithmic equation $\log_a x = b$.

This inverse relationship between logs and exponents is all you need to know in order to answer a logarithm question on the ACT. If you see $\log_x 16 = 4$, then you know that $x^4 = 16$. You will be able to use this second, more manageable mathematical expression to answer the question.

http://www.sparknotes.com/testprep/books/act/chapter2.rhtml